Objective probabilities, quantum counterfactuals, and the ABL rule

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Abstract

The ABL rule is derived as a tool of standard quantum mechanics. The ontological significance of the existence of objective probabilities is discussed. Objections by Kastner [preceding article] and others to counterfactual uses of the ABL rule are refuted. Metaphysical presumptions leading to such views as Kastner is defending in her Comment are examined and shown to be unwarranted.

1 INTRODUCTION

Following Mermin [1], I have characterized some of the probabilities that quantum mechanics allows us to calculate as being objective in the sense that they have nothing to do with ignorance—there is nothing for us to be ignorant of [2]. I have argued that the objective probabilities associated with attributions of results to unperformed measurements should be calculated according to the ABL rule, first derived by Aharonov, Bergmann, and Lebowitz [3]. Ruth Kastner [4, 5, 6] and others [7, 8, 9] have raised objections concerning the appropriateness of probability assignments to counterfactuals based on the ABL rule. In the present article I re-derive the ABL rule, clarify the meaning of objective probabilities and the significance of their existence, and respond to these objections.

In Sec. 2 both the Born rule and the ABL rule are derived from the standard, quantum-mechanical representation of contingent properties [10] as projection operators on a Hilbert space. Section 3 refutes the objections by Kastner and the other

authors to counterfactual uses of the ABL rule. Section 4 elucidates the significance of the existence of objective probabilities, and Section 5 is devoted to unearthing unwarranted metaphysical presumptions leading to such views as Kastner is defending in her Comment [4].

2 OBJECTIVE PROBABILITIES AND THE ABL RULE

Quantum mechanics is unequivocal about the probabilities it assigns to the possible results of possible measurements. If we represent the potentially attributable [11] properties of a system as projection operators on a Hilbert space $\mathcal{H}$, there is no ambiguity as to the form of the prior probability measure $p\left(q_i, t\right)$, which is based solely on properties possessed by the system *before* the time t [12, 13, 14]:

$$p\left(q_i, t\right) = \mathrm{Tr}[\mathbf{W}(t)\mathbf{P}_{q_i}].\tag{1}$$

As is well known, $\mathbf{W}$ is a unique density operator [that is, a unique self-adjoint, positive operator satisfying $\mathrm{Tr}(\mathbf{W}) = 1$ and $\mathbf{W}^2 \le \mathbf{W}$]. Tr signifies the trace defined by the formula $\mathrm{Tr}(\mathbf{X}) := \sum_i \langle i|\mathbf{X}|i\rangle$ for any orthonormal basis $\{|i\rangle\}$ in $\mathcal{H}$. If $\mathbf{W}^2 = \mathbf{W}$, $\mathbf{W}(t)$ projects on a one-dimensional subspace of $\mathcal{H}$ and thus is equivalent—apart from an irrelevant phase factor—to a "state" vector $|\psi(t)\rangle$. Such a "state" is said to be "pure," and the system is said to be "prepared" in it. With $\mathbf{W}(t) = |\psi(t)\rangle\langle\psi(t)|$ and $\mathbf{P}_{q_i} = |q_i\rangle\langle q_i|$, we obtain the familiar Born rule:

$$p\left(q_i, t\right) = \mathrm{Tr}[|\psi(t)\rangle\langle\psi(t)|\mathbf{P}_{q_i}] = \langle\psi(t)|\mathbf{P}_{q_i}|\psi(t)\rangle = |\langle\psi(t)|q_i\rangle|^2.\tag{2}$$

Applying the Born rule twice, we obtain the prior probability that a system with a prior probability measure $|a\rangle\langle a|$ will first be observed to have the property $|q_i\rangle\langle q_i|$ and then be found in possession of the property $|b\rangle\langle b|$:

$$p\left(q_i, b|a\right) = |\langle a|q_i\rangle\langle q_i|b\rangle|^2.\tag{3}$$

(For simplicity's sake we will assume that the Hamiltonian is zero between measurements.) Although readers familiar with the mathematical formalism of quantum mechanics are not likely to stumble over this expression, it involves a conceptual

transition that needs to be justified. When we ask for the probability that q_i, given a, the projection operator $|q_i\rangle\langle q_i|$ represents a potentially attributable property of the system. When we ask for the probability that b, given q_i, the same operator represents a probability measure. How do we get from a property to a probability measure?

If we start out with a probability measure $\mathbf{W}_1 = |a\rangle\langle a|$ and then find that the system has the property $\mathbf{P}$, we must update our probability measure accordingly. If the measurement that yields the property $\mathbf{P}$ is ideal—as is generally assumed when discussing interpretational issues,—the updated probability measure assigns probability zero to any property $\mathbf{P}'$ for which $\mathbf{P}\mathbf{P}' = 0$. Hence the "state" of the system "collapses"—not mysteriously but self-evidently—to the "state"

$$\mathbf{W}_2 = \frac{\mathbf{P}|a\rangle\langle a|\mathbf{P}}{\langle a|\mathbf{P}|a\rangle}. \tag{4}$$

The denominator ensures that the probability of the trivial property, represented by the identity operator $\mathbf{1}$, remains 1. If we put $|q_i\rangle\langle q_i|$ in place of $\mathbf{P}$, this reduces to $\mathbf{W}_2 = |q_i\rangle\langle q_i|$. Thus the updated probability measure is represented by the same operator as the property observed [15].

Next, we consider the probability that a system with a prior probability measure $|a\rangle\langle a|$ will be found in possession of property $|b\rangle\langle b|$ given that in the meantime Q is measured but regardless of the result of this measurement. This is obviously the sum of probabilities

$$p(b|a,Q) = \sum_j |\langle a|q_j\rangle\langle q_j|b\rangle|^2. \tag{5}$$

According to Bayes' theorem, the probability that the intervening measurement yields q_i given that the prior probability measure is $|a\rangle\langle a|$ and given that the final measurement yields b, then is

$$p(q_i|a,b) = \frac{p(q_i,b|a)}{p(b|a,Q)} = \frac{|\langle a|q_i\rangle\langle q_i|b\rangle|^2}{\sum_j |\langle a|q_j\rangle\langle q_j|b\rangle|^2}. \tag{6}$$

This is (one of the possible forms of) the ABL rule. Like the Born rule, it follows straight from the quantum-mechanical representation of contingent properties as projection operators on a Hilbert space.

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In principle, both rules have an objective as well as a subjective application. If Q is actually measured, both rules assign probabilities that are subjective inasmuch as they are based on probability measures that fail to take account of at least one relevant fact—the result of the measurement of Q. In order to be considered objective, a quantum-mechanical probability must be assigned on the basis of all relevant facts. If the Q measurement is actually made, the objective probabilities associated with its possible outcomes are trivially either zero or one. Thus both rules can assign nontrivial objective probabilities only if the Q measurement is not made. But this is not sufficient. Since Born probabilities take no account of (future) facts about the system's future properties, they can be considered objective only if there are no relevant such facts. This is hardly ever the case. Hence, in general, objective probabilities are calculated according to the ABL rule. We can drop the qualifying "in general" if we use the trivial property in place of $|b\rangle\langle b|$ if there never will be any facts about the system's future properties. In this case the ABL rule reduces to the Born rule:

$$\frac{|\langle a|q_i\rangle|^2 \langle q_i|\mathbf{1}|q_i\rangle}{\sum_j \langle a|q_j\rangle\langle q_j|a\rangle\langle q_j|\mathbf{1}|q_j\rangle} = \frac{|\langle a|q_i\rangle|^2}{\langle a|\left(\sum_j |q_j\rangle\langle q_j|\right)|a\rangle} = |\langle a|q_i\rangle|^2. \tag{7}$$

Thus objective probabilities are calculated according to the ABL rule, and they are assigned to contrary-to-fact conditionals, or counterfactuals, of the following general form:

(A) If a measurement of observable Q were performed on system S between an actual measurement yielding the result $|a\rangle\langle a|$ at time t_a and an actual measurement yielding the result $|b\rangle\langle b|$ at time t_b, but no measurement is actually performed between t_a and t_b, then the measurement of Q would yield q_i with probability $p(q_i|a,b)$.

For $p(q_i|a,b)$ to be objective, it is not enough that Q is not actually measured; it is necessary that *no* measurement is performed between t_a and t_b. If any other measurement M is performed during this time span, $p(q_i|a,b)$ is based on an incomplete set of facts—it does not take account of the result of M—and is therefore subjective. Note that the antecedent clause states not only that the Q measurement is made

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(counterfactually or in a possible world) but also that at the times t_a and t_b the respective results $|a\rangle\langle a|$ and $|b\rangle\langle b|$ are obtained (that is, they are as in the actual world). The observation that the result obtained at t_b might have been different had the Q measurement been actually made, is irrelevant to the truth of (A) inasmuch as it is based on an incomplete set of facts—it does not take into account the result obtained at t_b—while statement (A) is based on a complete set of facts, as it must be in order to assign objective probabilities. Probabilities are objective only if they are assigned to unperformed measurements, and only if the assignment is based on all relevant facts. The second "if" translates into the strong *ceteris paribus* clause that all measurement results obtained in the actual world are also obtained in the possible worlds considered by (A). These possible worlds differ from the actual world only in that they contain one extra measurement that is not made in the actual world.

3   REPLY TO KASTNER

Ruth Kastner [4, 5, 6] and others [7, 8, 9] have raised objections concerning the appropriateness of probability assignments to counterfactuals based on the ABL rule. The first objection I will address is this [6]: Since the Q measurement is not actually made, the following rule should be used instead of the ABL rule (6):

$$p_K(q_i|a,b) = \frac{p(q_i,b|a)}{p(b|a)} = \frac{|\langle a|q_i\rangle\langle q_i|b\rangle|^2}{|\langle a|b\rangle|^2} = \frac{|\langle a|q_i\rangle\langle q_i|b\rangle|^2}{|\sum_j\langle a|q_j\rangle\langle q_j|b\rangle|^2}. \tag{8}$$

This rule combines, according to Bayes' theorem, the probability $p(q_i,b|a)$ that a system with a prior probability measure $|a\rangle\langle a|$ will first be observed to have property $|q_i\rangle\langle q_i|$ and then be found in possession of property $|b\rangle\langle b|$, with the probability $p(b|a)$ that an equally "prepared" [16] system will be observed to have property $|b\rangle\langle b|$ given that in the meantime no measurement is made. In the numerator Kastner assumes that Q is measured, and in the denominator she assumes that between t_a and t_b no measurement takes place. Whereas in the denominator of the ABL rule (6) probabilities are added—the Q measurement is assumed to be made,—in the denominator of Kastner's rule (8) amplitudes are added, which entails that the Q measurement is not made. The ABL rule thus is consistent—it assumes throughout that the Q

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measurement is made,— while the same cannot be said of Kastner's rule.

A similar inconsistency mars arguments by Sharp and Shanks [7], Cohen [8], Miller [9], and Kastner [5, 6] purporting to prove the general invalidity of counterfactual uses of the ABL rule. These arguments have been refuted—cogently, in my opinion—by Vaidman [17], though Kastner [18], predictably, takes a different view. What these "proofs" purport to show is that the counterfactual use of the ABL rule yields results that are inconsistent with standard quantum mechanics. Specifically, it is claimed that this use entails the following equation:

$$
\begin{aligned}
p\left(q_j|a\right) &= p\left(q_j|a, b_1\right) p\left(b_1|a\right) + p\left(q_j|a, b_2\right) p\left(b_2|a\right) \\
&= \frac{p\left(q_j, b_1|a\right)}{p\left(b_1|a, Q\right)} p\left(b_1|a\right) + \frac{p\left(q_j, b_2|a\right)}{p\left(b_2|a, Q\right)} p\left(b_2|a\right).
\end{aligned} \tag{9}
$$

Since the final measurement of the observable B, assumed to have two eigenvalues b_1 and b_2, is actually made, the Born probability $p\left(q_j|a\right)$ of the outcome q_j of an intermediate measurement of Q is uncontroversially the sum of the probabilities $p\left(q_j, b_1|a\right)$ and $p\left(q_j, b_2|a\right)$. According to eq. (9) this is not always the case. It is the case if

$$
\begin{aligned}
p\left(b_i|a, Q\right) &= \sum_j |\langle a|q_j\rangle\langle q_j|b_i\rangle|^2 \\
&= \left|\sum_j \langle a|q_j\rangle\langle q_j|b_i\rangle\right|^2 = p\left(b_i|a\right).
\end{aligned} \tag{10}
$$

This holds if $Q = A$ (the observable measured at time t_a) or $Q = B$, or if for $j \neq k$,

$$
\Re\left(\langle q_k|a\rangle\langle a|q_j\rangle\langle q_j|b_i\rangle\langle b_i|q_k\rangle\right) = 0. \tag{11}
$$

Kastner states these conditions in Refs. [5] and [6]. It is then argued that since the counterfactual use of the ABL rule in eq. (9) generally leads to inconsistencies with standard quantum mechanics, this use is illegitimate unless one of those conditions is satisfied. However, what is illegitimate is not the counterfactual use of the ABL rule but the equation on which this conclusion is based. Like Kastner's rule (8), eq. (9) combines expressions that imply that the intervening measurement is made—namely, $p\left(q_j|a, b_1\right)$ and $p\left(q_j|a, b_2\right)$—with expressions that imply the contrary,

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namely, $p\left(b_1|a\right)$ and $p\left(b_2|a\right)$. Note that it would not improve matters if instead of the ABL probabilities $p\left(q_j|a, b_i\right)$ the probabilities $p_K(q_j|a, b_i)$ were used in eq. (9) [which would ensure that $p\left(q_j|a\right) = p\left(q_j, b_1|a\right) + p\left(q_j, b_2|a\right)$] since the probabilities $p_K(q_j|a, b_i)$ involve the same inconsistency. In order to be consistent we must assume throughout that the intervening measurement is made. This entails that instead of the probabilities $p\left(b_i|a\right)$ the probabilities $p\left(b_i|a, Q\right)$ must be used in eq. (9), which likewise ensures that $p\left(q_j|a\right) = p\left(q_j, b_1|a\right) + p\left(q_j, b_2|a\right)$.

One might perceive a contradiction between the counterfactual (A) and my insistence on the necessity of assuming consistently that the intervening measurement is made. But this apparent "contradiction" is the very nature of a counterfactual. Counterfactuals are statements about conceivable worlds that are different from the actual world in certain specified respects and like the actual world in all other relevant respects. The worlds considered by (A) are different from the actual world in that between t_a and t_b a measurement is made that is not made in the actual world; in all other respects they are like the actual world. In particular, the measurements at t_a and t_b have the specified outcomes. Since (A) is a statement about conceivable worlds in which the Q measurement is made, the probabilities it assigns to the possible outcomes of this measurement must have the same values as the probabilities that we would assign, on the same basis, to the possible outcomes of the same measurement, if this were actually made.

Kastner denies the validity of the counterfactual (A) also on the ground that it allegedly violates the requirement of cotenability. A counterfactual C refers to conceivable worlds that must satisfy two conditions: They must be different from the actual world in certain specified respects, and they must be like the actual world in all other relevant respects. If it turned out that there are no conceivable worlds that satisfy both conditions, C would fail to satisfy this crucial requirement.

Consider a conceivable world $\mathcal{W}_i$ in which the Q measurement yields an outcome q_i to which (A) assigns probability zero. (To each possible outcome q_k there corresponds a conceivable world $\mathcal{W}_k$.) $\mathcal{W}_i$ fails to satisfy the second condition inasmuch as it is unlike the actual world in another relevant respect: It does not obey the actual

physical laws. But this is something that we learn from (A) rather than something that invalidates (A). Statement (A) itself tells us that the conceivable world $\mathcal{W}_i$ is not a nomologically possible world inasmuch as under the specified conditions a measurement of Q would never yield the result q_i.

What Kastner has in mind is something more serious. According to the metalinguistic account of counterfactuals [19], invoked by Kastner [5], a counterfactual is true if its *antecedent* conjoined with *laws of nature* and statements of *background conditions* logically entails its consequent. The background conditions must be stable (that is, they must hold independently of the truth value of the antecedent). Kastner claims that (A) violates either the requirement of cotenability—the background conditions depend on the truth value of the antecedent—or the laws of nature.

Before addressing Kastner's argument I will show that both the stability of the background conditions and the laws of nature are in fact respected by (A). The antecedent is the statement that a measurement of observable Q is performed on system S between t_a and t_b. The background conditions consist in the observation (or the factually warranted possession) of the properties $|a\rangle\langle a|$ and $|b\rangle\langle b|$ at the respective times t_a and t_b, as well as in the absence of any measurement between t_a and t_b other than that specified in the antecedent. The relevant laws of nature are the principles of quantum mechanics. Being statistical laws, they enable us to assign probabilities to the possible results of measurements, and being universal laws that have never been found to conflict with experimental data, they allow us to apply them counterfactually—to assign probabilities to the possible results of unperformed measurements. These assignments can be made on the basis of all relevant facts about either the past or the future properties of S using the Born rule, or on the basis of all relevant facts about the past and future properties of S using the ABL rule. Since both rules are part of standard quantum mechanics, none of these assignments can conflict with standard quantum mechanics. Nor can they conflict with the required stability of background conditions. If obtaining $|a\rangle\langle a|$ at t_a and $|b\rangle\langle b|$ at t_b is nomologically possible without interposition of any measurement, the same is nomologically possible whenever a measurement is interposed [20]. The

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reason this is so is that the interposition of a measurement never decreases the probability of obtaining $|b\rangle\langle b|$ at t_b given $|a\rangle\langle a|$ at t_a:

$$p\left(b|a\right) = |\langle a|b\rangle|^2 = |\sum_k \langle a|q_k\rangle\langle q_k|b\rangle|^2 \leq \sum_k |\langle a|q_k\rangle\langle q_k|b\rangle|^2 = p\left(b|a, Q\right). \qquad (12)$$

Thus the background conditions are consistent with both the truth and the falsity of the antecedent. And given the necessity of assuming consistently that the intervening measurement is made, the antecedent, conjoined with the relevant laws of nature and the background condition statement, logically entails that the probability of obtaining the value q_i is as given by the ABL rule [21].

If Kastner reaches a different conclusion, it is because her understanding of the background conditions and/or of the relevant laws of nature is different from mine. That she thinks differently about the background conditions is obvious from her discussion [5] of the experiment considered by Sharp and Shanks [7]. In this experiment spin-$\frac{1}{2}$ particles are prepared at time t_1 with probability measure $|a_+\rangle$ and subjected at time t_2 to a measurement of their spin component along the **b** axis. We are accustomed to read $|a_+\rangle$ as "spin up along direction **a**", but what $|a_+\rangle$ really signifies depends on the time to which it refers. While at t_1 the system possesses the property represented by $|a_+\rangle\langle a_+|$, for $t > t_1$ the ket $|a_+\rangle$ or the density operator $\mathbf{W} = |a_+\rangle\langle a_+|$ represents a probability measure that says nothing about properties possessed at t; it only tells us that the prior probability of obtaining the result "spin up along direction **c**" at the time t is $\langle c_+|\mathbf{W}|c_+\rangle$. By the same token, if the final measurement indicates that the property represented by the operator $|b_+\rangle\langle b_+|$ is possessed at t_2, then this operator also represents the "time-reversed" density operator $\mathbf{W}'$ that yields the posterior probability [22] $\langle c_+|\mathbf{W}'|c_+\rangle$ of obtaining the result "spin up along direction **c**" at the time $t < t_2$.

Kastner, following Sharp and Shanks, states that the measurement at t_2 yields a mixture M consisting of two subensembles. Thus she considers an ensemble of measurements, performed on an ensemble of systems, rather than an individual measurement, for an individual measurement does not yield a *mixture* consisting of subensembles; it yields a *result*, in this case either the property represented by $|b_+\rangle\langle b_+|$ or the property represented by $|b_-\rangle\langle b_-|$. Thereafter Kastner states what she

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takes to be "the basic conceptual problem": "in considering a counterfactual measurement of the spin along **c** (observable σ_c) [at an intermediate time t] we must take into account all the effects of that measurement on the system. The measurement of the observable σ_c results in a change in the mixture M of post-selected ensembles" into a different mixture M'. Kastner goes on to say that "the mixture, M or M', obtaining at time t_2 ...must enter into the counterfactual calculation," and that "the characterization of the mixture obtaining at t_2 must be included in the background condition statement ..." Since this mixture depends on whether or not the σ_c measurement is performed at the time t, Kastner concludes that the statement of background conditions holding when the antecedent is false (no intervening measurement) becomes false when the antecedent holds, and is therefore not cotenable with the antecedent.

Kastner is led to this fallacious conclusion by conflating statements about ensembles with statements about individual systems. It is true that if we start with an ensemble of systems possessing property $|a_+\rangle\langle a_+|$ at time t_1, the result or effect of an ensemble of σ_b measurements performed at t_2 is a mixture M consisting of two subensembles, one containing systems possessing the property $|b_+\rangle\langle b_+|$, and another containing systems possessing the property $|b_-\rangle\langle b_-|$, while the result or effect of two ensembles of measurements, one of σ_c performed at time t and one of σ_b performed at time t_2, is a mixture M' consisting of four subensembles corresponding to that many combinations of possible measurement outcomes. But all this is irrelevant to the truth of (A) or the cotenability of its antecedent with its background conditions, for (A) is a statement about an individual system, not a statement about an ensemble. The *only* effect of the intervening measurement on an individual system, under the specified background conditions, is that at the time t it has either the property $|c_+\rangle\langle c_+|$ or the property $|c_-\rangle\langle c_-|$, neither of which it has if the intervening measurement is not made. The relevant question is not: Might the final measurement have a different outcome from the one it actually has if the σ_c measurement were performed? The relevant question is: Are the background conditions—$|a_+\rangle\langle a_+|$ possessed at t_1 and $|b_+\rangle\langle b_+|$ possessed at t_2—consistent with both the truth and the falsity of the

antecedent? The answer is affirmative, and this is sufficient for cotenability and hence for the legitimacy of (A).

In her Comment [4], Kastner introduces one Dr. X who asks himself the irrelevant question: "How might the data of my experiment have changed if I had made a measurement at time t that I did not, in fact, make?" Obviously, Dr. X might not have obtained the result $|b\rangle\langle b|$ at the time t_b. The counterfactual (A) addresses a different question in that it rules out this possibility. While Dr. X's question assumes the possession of property $|a\rangle\langle a|$ at time t_a, (A) in addition assumes the possession of property $|b\rangle\langle b|$ at time t_b. According to Kastner, one gets from the question asked by Dr. X to the question addressed by (A) by requiring the following: "If a measurement of observable Q had been performed, system S would (with certainty) have been pre- and post-selected with outcomes a and b as in the actual world." This requirement, so Kastner claims, correctly expresses the cotenability that is necessary for the consistency and the truth of (A). And since this requirement is obviously "not guaranteed to hold," she concludes that the counterfactual statement (A) fails [23].

By requiring that system S would *with certainty* have the properties a and b at the respective time t_a and t_b, Kastner introduces an element of nomological necessity that makes nonsense of (A). What she thereby refutes is not (A) but the following counterfactual (A′), which requires no refutation because it is patently false:

(A′) If a measurement of observable Q were performed on an ensemble of systems between an actually performed measurement of observable A at time t_a and an actually performed measurement of observable B at time t_b, but no measurement is actually made between t_a and t_b, then the measurement of Q would yield q_i with probability $p(q_i|a, b)$, and the measurements of A and B would (with certainty) yield the respective outcomes a and b."

The counterfactual (A) attributes a probability to a possible result of a possible measurement hypothetically performed at time t on an *individual system S* which (in the actual world) *happens* to possess the properties a and b at the respective times t_a and t_b. As has been explained at length in Ref. [2], nothing ever causes (i) a measurement, *qua* attempt to determine the value of some observable, to yield

a result or (ii) a measurement, *qua* successful determination of the value of some observable, to take place. In other words, the actual events or states of affairs that indicate the possession of a contingent property (by a system) or of a value (by an observable) are causal primaries, and this not in the sense that nothing ever causes a measurement to yield this particular value rather than that, but in the sense that nothing ever causes a measurement to be successful or to take place. (A causal primary is an event or state of affairs the occurrence or existence of which is not necessitated by any cause, antecedent or otherwise.) What the laws of quantum mechanics encapsulate is statistical correlations between causal primaries. If we take certain measurement results as given, we can use those laws to assign probabilities to the possible results of other measurements, which may or may not be performed. In particular, we can consider the possession of a (at time t_a) and of b (at time t_b) as given, and use the laws of quantum mechanics to assign probabilities to the possible results of a not actually performed intervening measurement. In so doing we do *not* assume that the occurrences of the measurements at t_a and t_b are necessitated by anything, let alone that the respective outcomes a and b are.

The only way to test probability assignments is to determine relative frequencies with the help of appropriately selected ensembles. In the case of (A), the appropriate ensemble is an ensemble of possible worlds in which system S is identically "prepared" or "pre-selected" as well as identically "retropared" or "post-selected," and in which the Q measurement is made. Although Kastner introduces one Dr. X[†] who is "associated with" all of those possible worlds, it is obvious that this ensemble is not empirically accessible. It is possible, however, to reproduce this ensemble in the actual world, by turning it into an ensemble of identically "prepared" and "retropared" copies of S. In order to render testable the probabilities that (A) assigns, we have no choice but to use an ensemble of pre- and post-selected systems. Yet the probabilities that (A) assigns are single-case probabilities; they are assigned to the possible outcomes of a single measurement on a single system that *happens* to possess property a at time t_a and property b at time t_b. The selection does not reflect any nomological constraint but merely serves to make those probabilities measurable.

Kastner is committed to denying that counterfactual probability assignments can be tested inasmuch as she considers the probabilities that can be measured (using pre- and post-selected ensembles) to differ quantitatively from the corresponding counterfactual probabilities. Accordingly, she considers statement (A) to be distinct from the following statement:

(B) In a possible world in which observable Q is measured at time t and system S yields outcomes $|a\rangle\langle a|$ and $|b\rangle\langle b|$ at times t_a and t_b, respectively, the probability of obtaining result q_i is given by $p\,(q_i|a,b)$.

To claim a quantitative difference between counterfactual and non-counterfactual assignments of quantum-mechanical probabilities or a significant difference between statements (A) and (B) is to misunderstand the meaning of counterfactual assignments of quantum-mechanical probabilities. Saying that $p\,(q_i|a,b)$ is the (objective) probability with which q_i *would be* obtained, given the outcomes $|a\rangle\langle a|$ and $|b\rangle\langle b|$ at the respective times t_a and t_b, is in all relevant respects exactly the same as saying that $p\,(q_i|a,b)$ is the (subjective) probability with which q_i *is* obtained given the same outcomes at times t_a and t_b. What else could statement (A) possibly mean?

None of the arguments marshaled by Kastner and the authors cited by her succeed in proving that statement (B) "differs significantly" from statement (A). That the outcome at t_b might be different from $|b\rangle\langle b|$ if the Q measurement were made, given the outcome at t_a alone, is irrelevant since the outcomes at t_a and t_b are both given. The argument from cotenability fails because it conflates statements about individual systems with statements about ensembles of systems. The attempt to replace $p\,(q_i|a,b)$ (eq. 6) by $p_K(q_i|a,b)$ (eq. 8) in (A) and the attempt to show that counterfactual uses of the ABL rule yield consequences that are inconsistent with quantum theory, both fail because they combine expressions that imply that the Q measurement is both performed and not performed. One does not do justice to the counterfactuality of probability assignments by arguments that begin by assuming that Q is not measured and end up by assuming that Q is measured. One does it justice by considering possible worlds in which Q is measured or a suitably pre- and post-selected ensemble of actual-world systems.

4 THE SIGNIFICANCE OF OBJECTIVE PROBABILITIES

In this section I want to elucidate the significance of the existence of objective probabilities. Such probabilities are not merely best guesses based on a complete knowledge, and thus free from any element of ignorance; they also tell us something important about the objective world.

To begin with, let us consider the following experiment [24]. A particle initially in possession of the property $|\psi_1\rangle\langle\psi_1|$ is eventually found in possession of the property $|\psi_2\rangle\langle\psi_2|$, where $|\psi_1\rangle = (|A\rangle + |B\rangle + |C\rangle)/\sqrt{3}$ and $|\psi_2\rangle = (|A\rangle + |B\rangle - |C\rangle)/\sqrt{3}$. The projection operators $\mathbf{P}_A = |A\rangle\langle A|$, $\mathbf{P}_B$, and $\mathbf{P}_C$ represent the respective properties of being inside one of three sealed boxes A, B, and C at an intermediate time t. The ABL probability of finding the particle inside box A at the time t is

$$p\left(A|\psi_1, \psi_2\right) = \frac{|\langle\psi_1|\mathbf{P}_A|\psi_2\rangle|^2}{\sum_j |\langle\psi_1|\mathbf{P}_j|\psi_2\rangle|^2}. \tag{13}$$

As it stands, $p\left(A|\psi_1, \psi_2\right)$ is underdetermined. To assign to it a value, we still have to specify exactly which observable is being measured. If it is the observable Q whose eigenkets are $|A\rangle$, $|B\rangle$, and $|C\rangle$, the denominator is given by

$$|\langle\psi_1|\mathbf{P}_A|\psi_2\rangle|^2 + |\langle\psi_1|\mathbf{P}_B|\psi_2\rangle|^2 + |\langle\psi_1|\mathbf{P}_C|\psi_2\rangle|^2 = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3}. \tag{14}$$

Since the numerator is equal to 1/9, $p\left(A|\psi_1, \psi_2\right)$ is equal to 1/3. If on the other hand we measure the binary observable $Q_A = \mathbf{P}_A$, the denominator is given by

$$|\langle\psi_1|\mathbf{P}_A|\psi_2\rangle|^2 + |\langle\psi_1|(\mathbf{P}_B + \mathbf{P}_C)|\psi_2\rangle|^2 = \frac{1}{9} + 0 = \frac{1}{9}, \tag{15}$$

and $p\left(A|\psi_1, \psi_2\right)$ is equal to 1. By the same token, if we measure Q then $p\left(B|\psi_1, \psi_2\right) = 1/3$, and if we measure Q_B then $p\left(B|\psi_1, \psi_2\right) = 1$.

I would like to discuss these results in the context of a somewhat more realistic setup. Consider a wall W in which there are three holes A, B and C. In front of the wall there is a particle source E. Behind the wall there is a particle detector D. Both E and D are equidistant from the three holes. Behind C there is a device that causes a phase shift by π. $\mathbf{P}_j$ now represents the alternative "the particle goes through hole

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j", where j may also stand for a union like $B \cup C$, the opening made up of B and C. For particles emitted by E the prior probability measure (with respect to the time at which they pass the wall) thus is $|\psi_1\rangle$, and for particles detected by D the posterior probability measure (with respect to the same time) is $|\psi_2\rangle$. To measure Q_A, we place near A a device F_A that beeps whenever a particle passes through A. To measure Q_B, we place near B a device F_B that beeps whenever a particle passes through B. To measure Q, we use both devices. (If the devices are 100% efficient, the absence of a beep then tells us that the particle went through C.)

What we just found is this: If only F_A is in place, the particle goes through A with probability one (assuming, of course, that it is both emitted by E and detected by D). If only F_B is in place, the particle goes through B with probability one. If both beepers are in place, the particle is equally likely to go through any of the three holes. Hence ABL probabilities in general are *contextual*—they depend on the distinctions that a particular setup permits us to make [25]. By measuring Q we can tell whether the particle goes through A, through B, or through C. By measuring Q_A, we can tell whether it goes through A or through $B \cup C$. With the former setup (both beepers in place) three properties are available for predication—"through A", "through B", and "through C",—with the latter (one beeper in place) only two are available. With the former setup three spatial distinctions are warranted—between A and B, between A and C, and between B and C,—with the latter only one is warranted. $p(A|\psi_1, \psi_2)$ depends on the spatial distinctions that are warranted within the complement of A in $A \cup B \cup C$. If none are warranted by the experimental setup then $p(A|\psi_1, \psi_2) = 1$. If the distinction between B and C is warranted by the setup then $p(A|\psi_1, \psi_2) = 1/3$.

The contextuality of objective ABL probabilities makes it obvious that probability one does not imply actual possession of a value (by an observable) or of a property (by a physical system): From our ability to infer with probability one the result of measuring a physical quantity at time t, it does not follow that at the time t there exists an element of reality corresponding to the physical quantity and having a value equal to the predicted measurement result. Hence the *only* reason we have for attributing a contingent property q to a physical system or a value v

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to a quantum-mechanical observable is the occurrence/existence of an actual event or state of affairs from which the possession of q or v can be inferred, or by which it is indicated. In other words, the contextuality of ABL probabilities implies that the contingent properties of physical systems are *extrinsic*: They supervene on what happens or is the case in the rest of the world. To paraphrase Wheeler [26], *no property is a possessed property unless it is an indicated property.* In short, owing to their contextuality, ABL probabilities cannot be assigned without specifying the range of values of an observable that is measured or assumed to be measured; and owing to their extrinsic nature, contingent properties cannot be attributed unless their possession is warranted by facts [27].

It may be held that, unlike an ABL probability equal to one, a Born probability equal to one is sufficient for the possession of a value [28], or that an element of reality corresponding to an eigenvalue of an observable Q exists at a time t if the Born probability measure for the time t has the pure form $|\psi(t)\rangle\langle\psi(t)|$ and $|\psi(t)\rangle$ is an eigenstate of Q. But this is an error. The extrinsic nature of contingent properties can be established without invoking ABL probabilities [2]. The contextuality of ABL probabilities merely confirms it [29].

Cohen [8] states that in standard quantum mechanics, probabilities for obtaining particular results are not contextual. What he means is that *Born* probabilities are not contextual. As was shown in Sec. 2, both the Born rule and the ABL rule follow straight from the quantum-mechanical representation of contingent properties as projection operators on a Hilbert space. Both therefore are tools of standard quantum mechanics. Cohen further states that the product rule is always valid in standard quantum mechanics. According to the product rule, if X and Y are commuting observables, if a measurement of X will yield x with Born probability one and a measurement of Y will yield y with Born probability one, then a measurement of XY will yield xy with Born probability one. This too is a statement about Born probabilities, not a statement about standard quantum mechanics. ABL probabilities do not always satisfy the product rule, and therefore standard quantum mechanics does not always satisfy the product rule. The operators $Q_A = \mathbf{P}_A$ and $Q_B = \mathbf{P}_B$

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commute; an intervening measurement of Q_A would yield 1 ("through A") with probability one; an intervening measurement of Q_B would yield 1 ("through B") with probability one; yet a measurement of $Q_A Q_B$ would yield nothing because (i) $Q_A Q_B = 0$ and (ii) there is no world in which $Q_A Q_B$ can be measured. While Q_A is measured in worlds in which *only* F_A is in place, Q_B is measured in worlds in which *only* F_B is in place. Hence it is *logically* impossible for both Q_A and Q_B to be measured in the same world. What is measured in a world in which both beepers are in place is not $Q_A Q_B$ but Q. Thus the product rule is "violated" only by combinations of counterfactual statements referring to different possible worlds.

Either type of probability has its specific use. The ABL rule is obviously of no use for *predicting*, on the basis of data obtained before time t, the result of a measurement performed at the time t, for its application presupposes knowledge of the results of measurements performed after the time t. [Even if the measurement at the time t is the last measurement ever to be performed on the system, we would have to know this in order to have sufficient information for applying the ABL rule (6) with the trivial property **1** in place of $|b\rangle\langle b|$.] For predictions, we must use the Born rule. The Born rule, on the other hand, is of little use when it comes to sounding the ontological implications of quantum mechanics, for these must be based on objective probabilities (which are free of any element of ignorance), and such probabilities, as was shown in Sec. 2, are ABL probabilities.

With Mermin [1] I believe that all the mysteries of quantum mechanics can be reduced to the single puzzle posed by the existence of objective probabilities. As I see it, the key to this puzzle is the contingent reality of spatial and temporal distinctions [2]. We are neurophysiologically disposed to think of space as something that exists by itself (rather than by virtue of the relational properties of matter), that contains matter (rather than spatial relations between material objects), and that is intrinsically and infinitely differentiated and thus adequately represented by a set of points that can be labeled by triplets of real numbers [30]. Yet if all conceivable spatial divisions were intrinsic to space, they would have an unconditional reality, and one of the following statements would necessarily be true of every object S contained

in the union $R \cup R'$ of two spatial regions: (i) S is inside R; (ii) S is inside R'; (iii) S has two parts, one in R and one in R'. No particle could ever pass through the union of two slits without passing through either slit in particular *and* without consisting of parts that pass through different slits. But this is precisely what particles do when interference fringes are observed in two-slit experiments (and we do not postulate hidden variables). Hence, spatial divisions cannot be intrinsic to space. They have a contingent reality. Like the contingent properties of quantum-mechanical systems, they are extrinsic; they supervene on the actual goings-on in the physical world, and they may be real for one object and nonexistent for another.

It is a fundamental principle of quantum mechanics that the probability of a process $\mathcal{P}$ capable of following several alternatives depends on whether or not the alternative taken by the process is indicated or capable of being indicated. If something indicates the alternative taken or if this is capable of being indicated, the probability of the process is given by the sum of the probabilities associated with its alternatives. By "capable of being indicated" I mean that the alternatives of $\mathcal{P}$ are correlated with the alternatives of another process $\mathcal{P}'$ such that a determination of the alternative taken by $\mathcal{P}'$ reveals the alternative taken by $\mathcal{P}$. (Paradigm examples of this kind of a situation are the experiments of Einstein, Podolsky, and Rosen [31, 32] and of Englert, Scully, and Walther [33, 34, 35].) On the other hand, if nothing either indicates or is capable of indicating the alternative taken by the process, the probability of the process is given by the absolute square of the sum of the amplitudes associated with the alternatives. What is the meaning of this fundamental principle?

I submit that if the alternative taken is indicated, we add probabilities *because* in this case the conceptual distinction that we make between the alternatives has a reality for the process or the system undergoing it—the distinction corresponds to something in the objective world. If the alternative taken is neither indicated nor capable of being indicated, we add amplitudes *because* in this case the conceptual distinction that we make between the alternatives has no reality for the process or the system undergoing it—the distinction corresponds to nothing in the objective world [36]. In our three-hole experiment it is the presence of F_A that makes the spatial

distinction between A and $B \cup C$ a reality for the particle. Assuming the beepers to be 100% efficient [37], the presence of F_A warrants two objective truth values, one for the proposition "The particle goes through A" and one for the proposition "The particle goes through $B \cup C$". The existence of these two truth values warrants the reality, for the particle, of the distinction between the two spatial regions A and $B \cup C$. The reality of this distinction is the reason why in the denominator of the right-hand side of eq. (13) we add the probabilities associated with the alternatives represented by $\mathbf{P}_A$ and $\mathbf{P}_{B \cup C} = \mathbf{P}_B + \mathbf{P}_C$ (eq. 15). By the same token, the presence of both beepers warrants objective truth values for three propositions, "The particle goes through A", "The particle goes through B", and "The particle goes through C". The existence of these three truth value warrants the reality, for the particle, of the distinction between the three spatial regions A, B and C. The reality of this distinction is the reason why in the denominator of the right-hand side of eq. (13) we add the probabilities associated with the alternatives represented by $\mathbf{P}_A$, $\mathbf{P}_B$, and $\mathbf{P}_C$ (eq. 14).

It needs to be stressed that this explanation of the uncertainty principle—as stated by Feynman and Hibbs [38]: Any determination of the alternative taken by a process capable of following more than one alternative destroys the interference between alternatives—owes nothing to the ABL rule. It rests on the fact that a successful measurement of an observable Q with n eigenvalues warrants attributing n truth values to n propositions, and thus warrants the objective distinctness of the n alternatives. The contextuality of ABL probabilities (as well as Born probabilities [29]) merely emphasizes the contingent reality of the distinctions we are wont to make. Given two observables with a common eigenvalue q, the ABL probability associated with q in general depends on which of the two observables is being measured—it depends on the entire spectrum of the observable being measured. In particular, the probability of finding that the particle goes through A depends on whether or not our distinction between B and C is objectively warranted (that is, whether or not the measurement is capable of distinguishing between "The particle goes through B" and "The particle goes through C").

The ontological significance of objective probabilities, then, is that they signal the unreality of some of the conceptual distinctions that we make. Since the warranted distinctions depend on what precisely is indicated, so do the probabilities of the corresponding alternatives.

If we conceptually partition space into smaller and smaller regions, we eventually arrive at a partition $\{R_i\}$ into *finite* (rather than infinitesimal) regions that are so small that the distinctions we make between them have no reality at all [2, 30]. Our spatial distinctions bottom out in a sea of objective probabilities. At a scale at which position-indicators ("detectors") with sufficiently small and sufficiently localized sensitive regions no longer exist, all we can say is counterfactual and probabilistic. This tells us that the world is only finitely differentiated spacewise. Conversely, the limited spatial differentiation of the objective world finds its proper expression in counterfactual assignments of objective probabilities.

What is true of the world's spatial aspect is equally true of its temporal aspect. There is no such thing as an intrinsically differentiated time, and therefore not only the contingent properties of things but also the times at which they are possessed are extrinsic. What is temporally differentiated is physical systems, and every physical system is temporally differentiated to the extent that it passes through successive states, in the proper sense of "state" that connotes properties indicated by facts. And since no finite system passes through an infinite number of successive states in a finite time span, no such system is infinitely differentiated timewise. The times that exist for a system S are the (factually warranted) times at which it has (factually warranted) properties [2, 30].

How, then, are we to conceive of system S during the interval between the times t_a and t_b? Since during this interval S lacks factually warranted properties, all that can be said about S between t_a and t_b is counterfactual and probabilistic. Our conceptual temporal distinctions, too, bottom out in a sea of objective probabilities. Not only is there no state (in the proper sense just defined) that obtains during this interval, but also there is no time between t_a and t_b at which any state could obtain.

The importance of this result cannot be overemphasized. It entails that the pa-

rameter t appearing in the (prior) Born probability $|\langle a(t)|q_i\rangle|^2$, in the posterior Born probability $|\langle q_i|b(t)\rangle|^2$, and in the ABL probability $|\langle a(t)|q_i\rangle\langle q_i|b(t)\rangle|^2 / \sum_j |\langle a(t)|q_j\rangle\langle q_j|b(t)\rangle|^2$ cannot be interpreted as the time at which anything obtains *per se*. The parameter t refers to the time of a measurement. Only if a measurement is actually made does it represent a time that exists for S because only then is there a contingent property that can be attributed to S at the time t. If the measurement is not actually made, the time at which it is made in a possible world does not exist for the actual-world edition of S. It follows in particular that neither the prior probability measure $|a(t)\rangle$ nor the posterior probability measure $|b(t)\rangle$ nor the ABL probability measure $\langle a(t)\|b(t)\rangle$ [39] can be interpreted as something that obtains at the time t [40]. If the measurement is made, what obtains at time t is a result rather than a probability measure, and if the measurement is not actually made, there is no time t at which anything concerning S could obtain.

5   QUANTUM COUNTERFACTUALS AND THE "FLOW" OF TIME

In this section I want to point out a common but unwarranted assumption about the temporal aspect of the physical world, and I want to show that this assumption leads to the views that Kastner is defending in her Comment [4]. Needless to say, I cannot avow that she actually makes this assumption.

The conclusions we have reached in the previous section run counter to a common way of thinking about time, according to which the experiential now and the temporal distinctions that we base on it are features of the physical world—the world accessible to physics. The experiential now is temporally unextended and undifferentiated. If it did correspond to something in the physical world, this would seem to warrant the notion of an objective *instantaneous* state that evolves in an infinitely differentiated time. Yet this contradicts the fact that the world is only finitely differentiated timewise.

In truth, nothing in the physical world corresponds to the experiential now and the temporal distinctions that we base on it. There simply is no objective way to

characterize the present or to distinguish between the past, the present, and the future. These distinctions can be characterized only subjectively, by how they relate to us: through memory, through the present-tense immediacy of qualia (introspectible properties like pink or turquoise), or through anticipation. In the world accessible to physics we may qualify events or states of affairs as past, present, or future *relative to* other events or states of affairs, but we cannot speak of *the* past, *the* present, or *the* future.

In classical physics this is not blindingly obvious. Classical physics is consistent with the notion of an instantaneous state that evolves in an infinitely differentiated time, and that encapsulates not only possessed properties but also everything that (i) happened or obtained at earlier times and (ii) is causally relevant to what happens or obtains at later times. This is how we come to conceive of "fields of force" that evolve in time (and therefore, in a relativistic world, according to the principle of local causality) and that causally link earlier times to later times (and therefore, in a relativistic world, local causes to their distant effects). But this does not entail that the notion of an *evolving* instantaneous state is itself consistent. If we conceive of temporal relations in the physical world, we conceive of their relata *at the same time* even though they happen or obtain *at different times*. Since we can't help it, that has to be OK. But it is definitely not OK if we introduce into our simultaneous and spatial mental picture of a temporal whole anything that evolves or advances across this temporal whole. One cannot represent a spatiotemporal whole as a simultaneous spatial whole and then imagine the present as advancing through it or an instantaneous state as evolving in it. To do this is to depict the spatiotemporal whole $\mathcal{U}$ as persisting unchanged in a time that is extraneous to $\mathcal{U}$, and to depict something as advancing or evolving across the unchanging $\mathcal{U}$ in that extraneous time. There is only one time, the fourth dimension of space-time. There is not another time in which anything evolves or advances across space-time as if space-time itself—rather than our mental picture of it—were a persisting and unchanging whole. If the present is anywhere in the spatiotemporal whole, it is trivially and vacuously everywhere—or, rather, everywhen.

To philosophers the perplexities and absurdities entailed by the notion of an advancing present or a flowing time are well known [41]. Physicists began to recognize the subjectivity of the present and the nonexistence of an *evolving* instantaneous state with the discovery of the relativity of simultaneity. In the well-known words of Hermann Weyl, "The objective world simply *is*; it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time." [42] Yet the non-objectivity of the now remains deeply counterintuitive [43]. Where space is concerned, we have no difficulty in abstracting from our subjective, perspectival point of view and adopt "the view from nowhere" [44]. Where time is concerned, we find it incomparably more difficult to abstract from our subjective, present-centered point of view and adopt "the view from nowhen" [45], not least because this seems to conflict with our incorrigible self-perception as free agents, which seems to require an open future [46].

In the world of physics, the future is as closed as the past. (This follows directly from the non-objectivity of the distinction between the future and the past.) If system S has property $|a\rangle\langle a|$ at time t_a and property $|b\rangle\langle b|$ at time t_b, then it always has been and always will be true that system S has these properties at the respective times t_a and t_b. There is nothing objectively or physically open about this. All that is "open" in respect of this is our knowledge, prior to these times, of the properties possessed at these times. What *is* (and always has been and always will be) objectively or physically open is the results of unperformed measurements. That is why we can assign to them objective probabilities.

If we retain the fallacious notion of the objectivity of the now and of the temporal distinctions that are based on it, or if we subscribe to the ensuing idea of an evolving instantaneous state, we are committed to regarding system S as infinitely differentiated timewise and to attributing to S a state that obtains at every instant of time. Quantum mechanics offers us not one but three candidates for such a state: the "prepared" or "retarded" state $|a(t)\rangle$, the "retropared" or "advanced" state $|b(t)\rangle$, and the time-symmetric two-state $\langle a(t)\|b(t)\rangle$. For reasons that are psy-

chological rather than physical, quantum realists usually settle for $|a(t)\rangle$, which leads to the well-known measurement problem. (The projection postulate is as mysterious when applied to evolving states of affairs as it is trivial when applied to probability measures.)

Another consequence of the myth of an evolving instantaneous state is that a difference in what obtains at the intermediate time t must make a difference to what obtains at later (earlier) times given that it makes no difference to what obtains at earlier (later) times. The intervening measurement "disturbs the system"—an ubiquitous but illegitimate phraseology found, for instance, in Sharp and Shanks [7],— with the result that the "disturbed" system is necessarily different from the "undisturbed" system either before or after the "disturbance" (or both). This notion implies an apparent infringement of cotenability: The state that obtains before the hypothetical measurement of Q and the state that obtains after this measurement cannot be both independent of the truth value of the antecedent of (A). As Kastner puts it, "holding fixed *both* pre- and post-selection states" [4] is impossible. If the prepared state $|a\rangle$ obtains before the Q measurement at time t then either $|a\rangle$ or one of the eigenstates of Q obtains after the time t, depending on whether or not the measurement is made. If the retropared state $|b\rangle$ obtains after the time t then either $|b\rangle$ or one of the eigenstates of Q obtains before the time t, depending on whether or not the measurement is made [47]. (If one thinks of $\langle a\|b\rangle$ as the state that obtains between t_a and t_b in the absence of an intervening measurement, an intervening measurement changes both the earlier and the later state: If the measurement yields q_i, the former state becomes $\langle a\|q_i\rangle$ while the latter state becomes $\langle q_i\|b\rangle$.)

The relevant background conditions, however, are not probability measures, nor are these states that obtain. The relevant background conditions are the possessed properties that determine probability measures. The unmeasured system differs from its possible-world counterpart neither at the times t_a and t_b (at which times the same properties are possessed in both worlds) nor during the intervals between t_a and t and between t and t_b (during which intervals no properties are possessed in either world) but only at the time t, which exists for the measured system but not for the

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unmeasured one. The intervening measurement has no influence whatsoever on what obtains at any other time. It has an influence on some probability measures but none on the *relevant* probability measures. The relevant probability measures are the prior probability measure $|a\rangle$ for times *earlier* than t and the posterior probability measure $|b\rangle$ for times *later* than t, while what is affected by the intervening measurement is the prior probability measure $|a\rangle$ for times *later* than t and the posterior probability measure $|b\rangle$ for times *earlier* than t. The ABL probability for the time t is obtained by combining, according to Bayes' theorem, the probabilities that are unaffected by the measurement, and thus it is independent of whether or not the measurement is actually performed.

Cohen [8] appears to reject the counterfactual use of the ABL rule not only on the basis of the intrinsically inconsistent eq. (9) but also on the ground that that use is "not consistent with realist interpretations of quantum mechanics." This, however, is no ground for rejecting counterfactual uses of the ABL rule. Rather, it is ground for rejecting realist interpretations of quantum mechanics. By "realist interpretations" Cohen may mean interpretations that endorse either Redhead's sufficiency condition [28], according to which Born probability one implies the existence of an element of reality, or the so-called "eigenstate-eigenvalue link," according to which "being in" an eigenstate of some observable implies the same. In point of fact, neither does imply the existence of an element of reality [2]. Or else Cohen may mean interpretations that construe probability measures as instantaneous states that evolve in an infinitely differentiated time. Such interpretations involve the double error of (i) treating time-dependent probability measures as if they were actual states and of (ii) extending temporal distinctions beyond the limits within which they are objectively warranted. Is instrumentalism the sole alternative to such realist interpretations? I submit that, on the contrary, the correct *ontological* interpretation of quantum mechanics can be found only when such realist interpretations are rejected. The highlights of that interpretation are the contingent reality of spatial and temporal distinctions and the existence of finite limits to the spatial and temporal differentiation of the physical world.

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[10] A contingent property of system S is a property that S may but does not necessarily (that is, by definition) possess. It corresponds to an eigenvalue of a quantum-mechanical observable on S.

[11] If possessed properties are attributable, properties capable of being possessed are "potentially attributable."

[12] J.M. Jauch, *Foundations of Quantum Mechanics* (Addison-Wesley, Reading, MA, 1968), pp. 92–94, 132.

[13] A. Cassinello and J.L. Sánchez-Gómez, "On the probabilistic postulate of quantum mechanics," Found. Phys. **26**, 1357–1374 (1996).

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[15] Much confusion can be avoided by maintaining a clear distinction between projection operators representing probability measures and projection operators representing potentially attributable properties. If a system with a vanishing Hamiltonian is observed to possess the property $|\psi\rangle\langle\psi|$ at the time t_0, the Born probabilities of the possible results of any measurement performed at a time $t > t_0$ are given by the probability measure $|\psi\rangle\langle\psi|$. Sloppiness about the distinction between probability measures and properties leads to the insidious notion that the system possesses the property $|\psi\rangle\langle\psi|$ not only at the time t_0, when it is factually warranted, but also at the time $t > t_0$, when the possession of this property is not warranted by any facts [2].

[16] The quotation signs serve as reminder that all that is actually prepared is a probability measure.

[17] Lev Vaidman, "Defending time-symmetrized quantum counterfactuals," Stud. Hist. Phil. Mod. Phys. **30**, 373–397 (1999).

[18] R.E. Kastner, "TSQT 'Elements of possibility'?," Stud. Hist. Phil. Mod. Phys. **30**, 399–402 (1999).

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[20] The converse is not true: If $\langle a|b\rangle = 0$, obtaining $|a\rangle\langle a|$ at t_a and $|b\rangle\langle b|$ at t_b is nomologically impossible unless another measurement (of an observable that has neither $|a\rangle$ nor $|b\rangle$ among its eigenkets) is interposed.

[21] A violation of cotenability is impossible for essentially the same reason that superluminal signaling is impossible. In the EPR-Bohm experiment (see Sec. XII of Ref. [2]) it is impossible for Bob not only to determine the spin component that is measured by Alice but also to find out whether or not any spin component is measured by Alice. The diachronic analogue of this experiment is a spin-1/2 particle on which Alice and Bob perform successive measurements. Instead of being performed in spacelike separation on a pair of particles, the measurements are performed in timelike separation on the same particle. And instead of being perfectly anticorrelated, the results of measurements of identical spin components are perfectly correlated. What remains unchanged is that it is impossible for Bob to deduce from the result of his measurement the spin component measured by Alice, or to find out whether any spin component is measured by Alice. But the possibility of obtaining a measurement outcome at time t_b from which one can infer whether or not the Q measurement is made at time t, is the *sine qua non* for a violation of cotenability. (Such a measurement outcome would be inconsistent with either the truth or the falsity of the antecedent.)

[22] Kastner uses the term "posterior probability" in a different sense, namely for the trivial probabilities (either zero or one) that are associated with the possible results of an actually performed measurement when the actual result is taken into account.

[23] Kastner also maintains that statement (A) is "a new type of counterfactual claim." If the "old type" is limited to such counterfactuals as may be formulated in the context of a classical theory, the two types do indeed differ significantly.

Since in classical physics all observables have definite values at all times, the antecedent of a classical counterfactual states that a specified observable has a value that is different from the value it actually has. The antecedent of the quantum counterfactual (A), by contrast, implies that a specified observable has a value while actually it has no value at all. While a classical counterfactual assumes that something obtains whereas in reality *something else* obtains, a quantum counterfactual assumes that something obtains whereas in reality *nothing* obtains. A classical counterfactual ignores an actually possessed property of the system to which it refers. The same is not true of a quantum counterfactual. This warrants the characterization of quantum counterfactuals like (A) and classical counterfactuals as being objective and subjective, respectively.

[24] Lev Vaidman, "Weak-measurement elements of reality," Found. Phys. **26**, 895–906 (1996).

[25] D.Z. Albert, Y. Aharonov, and S. D'Amato ("Curious new statistical prediction of quantum mechanics," Phys. Rev. Lett. **54**, 5–7, 1985) were the first to use the ABL rule to demonstrate the contextuality "of quantum-mechanical systems." Their argument was rejected by J. Bub and H. Brown ("Curious properties of quantum ensembles which have been both preselected and postselected," Phys. Rev. Lett. **56**, 2337–2340, 1986) on the grounds that Albert et al. employ statistical ensembles that are not well defined. Subsequently the conclusions of Bub and Brown were invalidated by W.D. Sharp and N. Shanks ("The curious quantum statistics in the interval between measurements," Phys. Lett. A **138**, 451–453, 1989) who showed that the ensembles employed by Albert et al. *are* well defined.

[26] "No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon."— John Archibald Wheeler, "Law without law," in *Quantum Theory and Measurement*, Ref. [3], pp. 182–213. See also John Archibald Wheeler, "On recognizing 'law without law' (Oersted Medal Response at the joint APS-AAPT Meeting, New York, 25 January 1983)," Am. J. Phys. **51**, 398–404 (1983).

[27] Albert et al. (Ref. [25]) have argued that in the interval between two measurements quantum-mechanical systems may have dispersion-free values for noncommuting observables, contrary to the Heisenberg uncertainty relations as standardly interpreted. In point of fact, quantum-mechanical systems have values only for *measured* observables. Since noncommuting observables cannot be measured simultaneously, they cannot simultaneously possess values.

[28] Michael Redhead, *Incompleteness, Nonlocality and Realism* (Clarendon, Oxford, 1987), p. 72.

[29] Nor is the contextuality of quantum-mechanical probability assignments restricted to probability assignments based on initial and final measurements. See, for instance, Asher Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic Publishers, Dordrecht, 1995), Chap. 7.

[30] Ulrich Mohrhoff, "Quantum mechanics and the cookie cutter paradigm," e-Print archive, http://xxx.lanl.gov/abs/quant-ph/0009001.

[31] Albert Einstein, Boris Podolsky, and Nathan Rosen, "Can quantum-mechanical description of physical reality be considered complete?," Phys. Rev. **47**, 777–780 (1935); reprinted in Wheeler and Zurek (Ref. [3]), pp. 138–141.

[32] David Bohm, *Quantum Theory* (Prentice Hall, Englewood Cliffs, NJ, 1951).

[33] Marlan O. Scully, Berthold-Georg Englert and Herbert Walther, "Quantum optical tests of complementarity," Nature **351**, No. 6322, 111–116 (1991).

[34] Berthold-Georg Englert, Marlan O. Scully and Herbert Walther, "The duality in matter and light," Scientific American **271**, No. 6, 56–61 (December 1994).

[35] Ulrich Mohrhoff, "Objectivity, retrocausation, and the experiment of Englert, Scully and Walther," Am. J. Phys. **67**, 330–335 (1999).

[36] There remains the question of why we add probabilities if the alternative taken is not indicated but capable of being indicated. If a process is capable of following one set $\{\mathcal{A}_k^1\}$ of mutually contradictory alternatives, it is in principle capable of following at least one other set $\{\mathcal{A}_k^2\}$ of mutually contradictory alternatives

such that the two sets are complementary. In the experiment of Englert, Scully, and Walther, for instance, experimenters may determine either the slit taken by an atom (by identifying the microwave cavity containing the photon that is left behind by the atom) or the phase relation with which the atom emerges from the slit plate (by opening the electro-optic shutters and watching the photosensor situated between the cavities). If the experimenters determine neither the slit taken nor the phase relation, then not only the conceptual distinctions we make between the alternatives of each set correspond to nothing in the objective world, but also the conceptual distinction we make between the two sets has no counterpart in the objective world. In this case we cannot add the amplitudes associated with either set of alternatives, for the amplitudes associated with one set are undefined unless the process takes a particular alternative from the complementary set. In particular, the amplitudes associated with the first set of alternatives (the atom takes this or that slit) are undefined unless the atom takes both slits with a particular phase relation.

[37] Even when we measure prior Born probabilities, we postselect: We discard measurements that did not succeed. Our euphemism for this procedure is that we use apparatuses that are 100% efficient. Such apparatuses are physically unrealizable on principle [2].

[38] R.P. Feynman, A.R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, NY, 1965).

[39] The symbol $\langle a\|b\rangle$ represents the "two-state" introduced by Yakir Aharonov and Lev Vaidman, "Complete description of a quantum system at a given time," J. Phys. A **24**, 2315–2328 (1991).

[40] This ought to be self-evident. The probability of finding the particle in box A at time t is not something that obtains at the time t, anymore than it is something that exists inside box A.

[41] See for instance the illuminating entry on "time" in *The Cambridge Dictionary of Philosophy* (Cambridge University Press, Cambridge, 1995), edited by Robert

Audi.

[42] Hermann Weyl, *Philosophy of Mathematics and Natural Science* (Princeton University Press, Princeton, N.J., 1949), p. 116.

[43] In the past I myself have, in increasingly moderate terms, argued for the objectivity of the temporal modes past, present, and future. I now consider it more fruitful if in our attempts to understand the larger world that includes consciousness we face up squarely to the subjectivity of the temporal modes.

[44] Thomas Nagel, *The View from Nowhere* (Oxford University Press, New York, NY, 1986).

[45] Huw Price, *Time's Arrow & Archimedes' Point* (Oxford University Press, New York, NY, 1996).

[46] This conflict is spurious. As was pointed out in Ref. [2], even if the future "already" existed in some way, my present choice can be one of its determining factors. What would be inconsistent with my self-perception as a free agent would be the possibility of knowing the future (and hence of knowing my choice before it was made).

[47] Kastner avows that her views are consistent with the time symmetry of the relevant physical laws, according to which reversing the direction of time flow changes one nomologically possible story into another such story. The time symmetry of the situation considered by statement (A) is of a different kind. It consists in the nonexistence of *any* direction of time flow. Where there is no advancing present and no evolving instantaneous state, there time does not flow.